A model describing stable coherent synchrotron radiation in storage rings

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We present a model for generating high power stable broadband coherent synchrotron radiation (CSR) in the terahertz frequency region in an electron storage ring. The model includes distortion of bunch shape from the synchrotron radiation, which enhances higher frequency coherent emission, and limits to stable emission due to a microbunching instability excited by the CSR. We use this model to quantitatively explain several features of the recent observations of CSR at the BESSY II storage ring. We also use this model to optimize the performance of a source for CSR emission.

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Coherent synchrotron radiation (CSR) occurs when the electrons in a bunch emit synchrotron radiation (SR) in phase. Coherent radiation intensity is proportional to the square of the number of particles per bunch in contrast to the linear dependence of the usual incoherent radiation. Since the number of particles per bunch is typically very large ($\gtrsim 10^6$), the potential intensity gain for a CSR source is huge. Although the possibility of steady state CSR in circular accelerators was discussed over 50 years ago [1], it is only recently that it has been observed for the first time at the BESSY II storage ring [2, 3]. As is illustrated by the first successful application of such a source [4], there is the exciting possibility of using this radiation as an innovative and powerful source in the terahertz frequency range. During the BESSY II measurements, the storage ring was tuned into a special mode with much lower momentum compaction than in the normal operation. Attractive features of the radiation were a flux increase of about five orders of magnitude over the incoherent synchrotron spectrum, a broadband frequency range from about 3 cm^{-1} to 60 cm^{-1} and the stability of the source. Several interesting properties of the radiation were also observed. First, the coherent emission spectrum extended to much shorter wavelengths than expected from a Gaussian electron bunch of the measured length. Second, a significantly non-Gaussian longitudinal distribution of the electron bunch was measured [5]. Third, a threshold beam current was observed, above which the CSR emission became unstable, generating radiation bursts at terahertz frequencies.

We have developed a model that for the first time accounts for the above-mentioned observations and provides a novel scheme for predicting and optimizing the performance of a ring-based CSR source. This letter presents the elements of the model, compares its prediction with the BESSY II observations and describes the design criteria for an optimized stable ring-based CSR source. Only the case of a bending magnet source point is considered.

The SR power spectrum is given by [1, 6]:

$$\frac{dP}{d\lambda} = \frac{dp}{d\lambda}[N + N(N-1)g(\lambda)] \tag{1}$$

where λ is the wavelength of the radiation, p is the single particle radiated power, N is the number of particles per bunch and g is the CSR form factor, the absolute square of the Fourier transform of the normalized bunch distribution. Here $dp/d\lambda$ is defined to account for shielding due to the conductive vacuum chamber. The shielding effect has been studied by several authors over many years [1, 7–10]. A salient feature is that $dp/d\lambda$ drops off abruptly for λ greater than the shielding cutoff wavelength λ_0, which is estimated to be about $2h\,(h/\rho)^{1/2}$, where h is the chamber height and ρ is the dipole bending radius. The first term in Eq. (1), linear in N, is the incoherent component of the power. The second term, proportional to $N^2 g(\lambda)$, represents a potentially much larger coherent component. Thus, to have significant CSR at the wavelength of interest we must have $g(\lambda) > 1/N$ and also $\lambda < \lambda_0$. For the particular case of a Gaussian distribution, CSR occurs for $\sigma_z \lesssim 2\lambda/\pi$ and $\lambda < \lambda_0$, with

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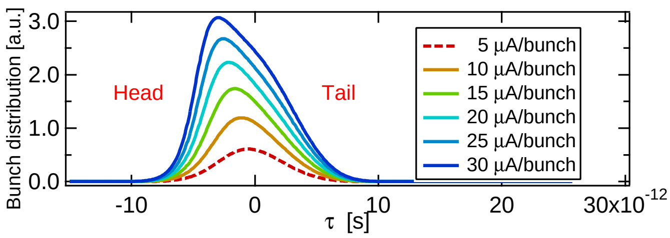

FIG. 1: Calculated equilibrium longitudinal distribution for different currents per bunch using the shielded SR wake. BESSY II case with a natural bunch length of 2.5 ps.

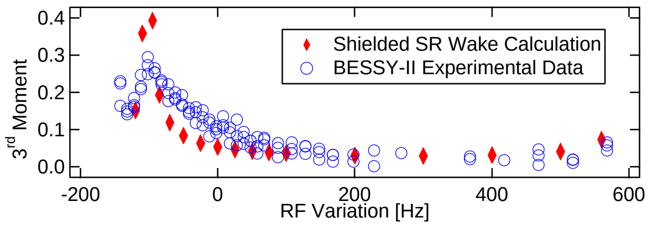

FIG. 2: Measured and calculated 3^{rd} moment of the longitudinal distribution of the BESSY II beam as a function of different radio frequency values.

σ_z the rms bunch length. In the general case, comparing at short wavelengths bunches with same rms length, non-Gaussian distributions yield a g factor significantly larger than Gaussian ones. This implies that these 'distorted' bunches can have CSR emission at wavelengths remarkably shorter than in the purely Gaussian case. We have found that the BESSY II results show this pattern. In fact, streak camera measurements [11] indicated bunch lengths of ~ 1 mm while CSR was clearly visible down to ~ 200 μm. The same streak camera measurements also showed clearly asymmetric distributions with the leading edge sharper than the trailing one.

Low current equilibrium longitudinal distributions in electron storage rings are usually Gaussian. Phenomena that can generate bunch distortions can be classified in two main categories, nonlinear dynamics and collective effects. RF and lattice nonlinearities belong to the first group, while SR and vacuum chamber wakes fall in the second one. For most storage rings, including BESSY II, RF nonlinearities are very small and can be neglected and our simulations showed that the distortion effects due to lattice nonlinearities are negligible. In the analysis of the collective effects we start with the ones induced by the SR, using for the SR wake the analytical expressions where the vacuum chamber shielding is represented by the parallel plates model [7–10].

To find the equilibrium longitudinal bunch distribution $I(\tau)$ in the presence of wakes we solve the Haïssinski equation [12]:

$$I(\tau) = Kexp\left[-\frac{(c\tau)^2}{2\sigma_{z0}^2} - \frac{c^2}{\sigma_{z0}^2 \dot{V}_{RF}} \int_{-\infty}^{\infty} I(\tau - t)S(t)dt\right] \tag{2}$$

where σ_{z0} is the natural bunch length, c is the speed of light, τ is the distance in time from the synchronous particle, $\dot{V}_{RF}$ is the time derivative of the radio frequency (RF) voltage at the synchronous phase, K is a normalization constant with the dimension of a current and $S(\tau)$ is the wake in the step response shape.

Figure 1 shows, for the case of BESSY II, the equilibrium longitudinal bunch profile calculated by solving Eq. (2) numerically for different currents per bunch and using the shielded SR wake. The results show several interesting features: with growing current per bunch, the distribution leans forward with an increasingly sharp leading edge, the bunch rear becomes less steep, and the bunch centroid shifts to earlier times (synchronous phase shift). The steepening of the bunch leading edge can be understood as the focussing effect due to the large gradient that the SR wake presents to the head of the bunch.

To confront predictions of the SR wake model with the observed bunch asymmetries at BESSY II we compare the third moment of the longitudinal distribution for the calculated and measured cases. In Fig. 2. the circles show the third moment of the streak camera measured distributions for different RF values at a constant very small current per bunch, $\sim 2\mu$A. In this special lattice configuration, the momentum compaction cannot be considered constant anymore and its dependence on the beam energy must be taken into account. Changes in the RF frequency modifying the energy induce at the same time variations in the momentum compaction and during the experiment in Fig. 2, this quantity ranged from $\sim 2 \times 10^{-6}$ to $\sim 7 \times 10^{-5}$. The interesting experimental observation was that the smaller the momentum compaction the larger was the third moment and consequently the asymmetry of the distribution. The diamonds in the figure show the third moment values calculated using the shielded SR model for the wake. The reasonable overall agreement with the measured values outlines the important result that the SR wakefield is able to explain the amplitude of the asymmetries observed in the bunch distribution. In comparing the data in Fig. 2 a constant offset has been added to the experimental values in order to equalize the minima of the two sets of points. In fact, in the third moment minimum region, the higher momentum compaction and the extremely low current per bunch generate distributions close to Gaussian with almost zero third moment. The offset in the experimental points can be attributed to systematic errors in the measurement.

For given current and wavenumber, we define the CSR gain as the ratio between the radiation intensities when CSR is present and when the emission is completely incoherent. Figure 3 shows the CSR gain measured at BESSY II and the calculated distributions as a function of the radiation wavenumber for two different currents per bunch. Also shown is the calculated CSR gain for

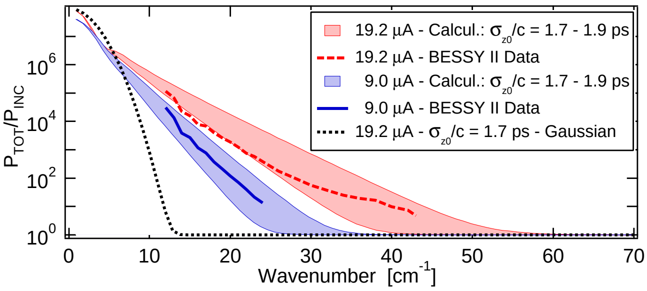

FIG. 3: CSR gain as a function of the wavenumber $1/\lambda$. The BESSY II data for two different currents per bunch are compared with the shielded SR calculation and with the curve for a Gaussian distribution of the same length.

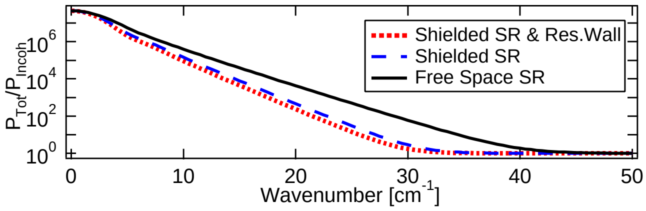

FIG. 4: CSR gain vs. radiation wavenumber calculated using the shielded SR wake with (dotted line) and without (dashed line) the resistive wall wake. The solid line shows the gain calculated with the free space SR wake. Case of BESSY II with 9.0 μA per bunch and 1.7 ps natural bunch length.

the undistorted Gaussian distribution. The colored areas show the shielded SR calculations obtained by varying the natural bunch length over a 10% range. This choice can be explained as follows. The natural bunch length used as input parameter for the simulations was derived from measurements of the synchrotron frequency, of the RF voltage and of other machine quantities. The experimental error for this evaluation is consistent with the 10% assumption. The comparison in Fig. 3 shows the general good agreement between calculations and data and also the strong power enhancement at the higher wavenumbers that the distorted case presents with respect to the Gaussian one. The data at higher current show a more irregular shape compared with the calculations. Possible factors that can contribute to this are systematic errors in the measurements and additional collective effects induced by vacuum chamber wakes.

In this last category, we have investigated the effect of the resistive wall (RW) wakefield [13, 14]. Figure 4 shows, for a particular case of BESSY II, the comparison between the CSR gain curves calculated using the shielded SR wake with (dotted line) and without (dashed line) the inclusion of the RW wake. The effect of the RW wake is clearly very small and slightly decreases the CSR gain. Additional calculations using a broad-band resonator model for the BESSY II vacuum chamber impedance showed a negligible contribution from this term. The solid line in Fig. 4 shows the CSR gain calculated using the free space (FS) SR wake [9, 10] for the interesting case where the vacuum chamber shielding is negligible. Compared to the FS case, the vacuum chamber shielding reduces the gain significantly, pointing out the important result that for maximizing the CSR gain in an optimized source the shielding effect must be kept negligible. For the case of the parallel plates model and Gaussian bunches, a simple criterion was derived [10]:

$$\Sigma = \frac{\sigma_z}{h}\left(\frac{2\rho}{h}\right)^{1/2} \lesssim 0.2 \qquad (3)$$

If $\Sigma \lesssim 0.2$, then the shielding effect is negligible and the FS SR wake can be used (note that $\Sigma \propto \sigma_z/\lambda_0$). Two examples: in the case of BESSY II with $\sigma_z \sim 1$ mm, $\rho = 4.35$ m and $h = 3.5$ cm, $\Sigma \sim 0.45$ and the shielding effect is relevant, while in a hypothetical but realistic CSR source with $\sigma_z = 300$ μm, $h = 4$ cm and $\rho = 1.33$ m, $\Sigma \sim 0.06$ and the shielding is negligible.

So far we have considered only static distortions to the bunch distribution leading to increased coherent emission. It is also possible for dynamic modulations of the distribution, or microbunching, to influence the emitted spectrum. A CSR-driven microbunching instability has been theoretically predicted [15], simulated [16] and experimentally verified [17, 18]. Above a current threshold, modulations in the current are amplified and appear as CSR bursts at terahertz frequencies. For a ring-based source of CSR, stability is a fundamental requirement for most applications. In the following, we show how to adjust the parameters of a ring such that the power and bandwidth are maximized while remaining below the threshold of the microbunching instability.

Assuming that criterion (3) is fulfilled, we can use the FS SR wake and following the approach used in ref. [19] we express the bunch population N as:

$$N = A\,(B/E)^{1/3} f_{RF} V_{RF} \sigma_{z0}^{7/3}\, F(\kappa)\ , \qquad (4)$$

where $A = 6.068\ 10^{-4}$ [SI units], B is the dipole magnet magnetic field, f_{RF} is the storage ring RF, V_{RF} is the RF peak voltage, and E is the beam energy. The numerical factor $F(\kappa) = \int y(x)dx$ is the integral of the solution of the equilibrium equation:

$$y(x) = \kappa \exp\left[-x^2/2 + \mathrm{sgn}(\alpha)\int_0^\infty y(x-z)z^{-1/3}dz\right]\ , \qquad (5)$$

which is a dimensionless form of Eq.(2) for the FS wake case, with $x = c\tau/\sigma_{z0}$. The factor $\mathrm{sgn}(\alpha)$ is the sign of the momentum compaction. The advantage of F is that it depends only on the dimensionless parameter κ. As κ increases, F, N, and the bunch distortion all increase. Using Eq. (1) for $Ng(\lambda) \gg 1$ and the expression for $dp/d\lambda$ when the wavelength is shorter than λ_0 but much larger than the SR critical wavelength (see for example [20]) we can write the power spectrum for a ring of length L and

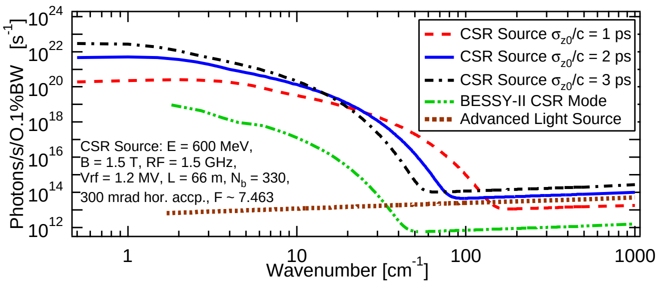

FIG. 5: Example of source optimized for the CSR production using the criteria described in this letter. The photon flux is compared with the cases of a conventional SR source (ALS) and of BESSY II CSR mode with 400 bunches, 19.2 μA/bunch, σ_{z0}/c= 1.8 ps and 60 mrad horizontal acceptance.

N_b bunches as:

$$\frac{dP}{d\lambda} = \frac{CN_b}{L}(f_{RF}V_{RF})^2 \left(\frac{B}{E}\right)^{1/3} \left(\frac{\sigma_{z0}^2}{\lambda}\right)^{7/3} F(\kappa)^2 g(\lambda) \ , \tag{6}$$

where $C = 2.642\ 10^{-21}$ [SI units].

To optimize the intensity and spectral bandwidth given by (6), we must first be sure that the bunch population is below the threshold for the previously mentioned microbunching instability, which is to say [15, 17]:

$$N \leq D\,(B/E)^{1/3} f_{RF} V_{RF} \sigma_{z0}^3 \lambda^{-2/3} \ , \tag{7}$$

where $D = 4.528\ 10^{-3}$[SI units]. By combining Eq. (4) and Eq. (7) the following stability criterion is derived:

$$F \leq F_{max} = G\,(\sigma_{z0}/\lambda)^{2/3} \ , \tag{8}$$

where $G = 7.463$ is a dimensionless constant. It must be remarked that the instability theory was derived for the case of a coasting beam. Anyway, simulations and experimental results at the ALS and at BESSY II [17, 18] showed that the model works also for bunched beams and that the theory is able to predict the instability threshold when in Eq. (7) $\lambda \sim \sigma_{z0}$. By (8) the corresponding threshold for F is $F_{max} \sim G$. The value of κ for maximum F is obtained by solving (5), increasing κ to a value κ_{max} such that $F(\kappa_{max}) = G$.

Now let us put $F = G$ in Eq.(6) and examine the remaining factors. Expressing g in terms of y, we find that $g(\lambda)$ is a function only of σ_{z0}/λ and κ (the latter now set equal to κ_{max}). Thus, the spectrum extends to larger wave numbers as σ_{z0} is decreased. On the other hand, the factor $\sigma_{z0}^{14/3}$ sharply degrades the overall intensity if σ_{z0} is too small. By an appropriate compromise in the choice of σ_{z0} we get a suitable spectral bandwidth. Once that choice is made, we can still vary the other factors in (6) to maximize radiation intensity while respecting technical constraints. It must be remarked that the momentum compaction, which does not appear explicitly in Eq. (6), is used in this scheme for keeping constant σ_{z0} when the other quantities are varied.

Figure 5 shows an example of the impressive performances that a source designed with the presented criteria can achieve. Also shown are the curves for a 'conventional' SR source (ALS) and for BESSY II in the special CSR mode.

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